Project Title: Hydroelastic response of large floating structures and arbitrarily shaped bodies in an environment of varying 3D bathymetry.

The study of interaction of free-surface gravity waves with floating deformable bodies is a very interesting problem which finds significant applications in hydrodynamic analysis and design of very large floating structures (VLFS, megafloats) operating offshore (as power stations/mining and storage/transfer), but also in coastal areas (as floating airports, floating docks, residence/entertainment facilities). In the same category of technological problems also belong issues relating to the construction and installation of floating bridges, floating marinas and breakwaters etc. For all the above problems hydroelastic effects are significant and should be properly taken into account. Extended surveys, including a literature review, have been presented by Kashiwagi (2000), Watanabe et al (2004). In addition, the effect of water waves on floating deformable bodies is also related to environmental issues finding important applications. A specific example concerns the interaction of waves with thin sheets of sea ice, which is particularly important in the Marginal Ice Zone (MIZ) in the Antarctic, a region consisting of loose or packed ice floes situated between the ocean and the shore sea ice (Porter & Porter, 2004). As the ice sheets support flexural–gravity waves, the energy carried by the ocean waves is capable of propagating far into the MIZ, contributing to break and melting of ice glaciers (Squire et al 1995, Squire 2007) and accelerating global warming effects and rise in sea water level.

Large floating marine/coastal structures: The rapid growth of population in urban development areas with tight housing space available, as well as in countries with a large number of islands (or with vast coastline), drive town planners and engineers to use the reclamation of land from the sea, in order to relieve congested cities. Creating areas of land in the sea by filling is applicable only when the water is shallow (less than 20 meters). However, when the depth of water is too large and the bed is not rigid enough, the landfill of the coastal area is no longer worthwhile economically. In addition, this technique causes ecological disasters in the marine environment. When faced with these physical limitations and environmental effects, very large floating structures (or mega-floats) are an alternative to creating a solid ground for operation.

The term mega-float or VLFS (Very Large Floating Structures) is used to describe a system that includes a floating structure, the docking and the access system (Figs.1, 2 and 3). Unlike vessels, mega-floats are very large floating surfaces, hence they are not likely to tumble, and are developed by linking the required number of floating construction units. The interior of the floating structure is made of separate water resistant compartments to avoid any leak, ensuring that even if one compartment fails the adjacent compartments will provide the necessary flotation and buoyancy for the whole structure.

In the case of mega-floats, the excessive horizontal dimensions, compared to the typical field wavelengths, lead to significant elastic displacement across the body. At a first-order of approximation, the structure is modeled as a thin floating elastic plate. The structure's dynamic deflection is characterised primarily by the vertical elastic displacement, caused by the surface waves, which also propagates from one structure edge to the other; (Fig. 4). This movement (which corresponds to an arrow bending) provides the basis for the calculation of loads and motions and is taken into account in the design and construction of the mega-float in order to ensure its proper functioning. It also provides basic information for further local dynamic analysis, study of the floating structure's strength and the material's fatigue, so that long term use of Mega-Floats to be guaranteed.

Figure 1. Floating airport (left). Floating island with touristic & recreational support installations (right) in Onomichi, Hiroshima, Japan.

In the near shore area, wave-structure interactions are also affected by the seabed. Indeed, surface gravity waves that propagate in relatively shallow regions feel the bed and are subject to shoaling, refraction and diffraction phenomena. In this case, the description of wave motion becomes more difficult as the bottom topography becomes more complex. These wave transformations have immediate effect to the wave-structure interaction and should be therefore taken into account when designing floating structures. In addition, there are side-effects associated with the development and/or transformation of local currents in the coastal zone around the structure which change the distribution of oxygen in the water column. The latter sometimes affecting significantly the marine biology of the region having serious environmental effects. All of the above make the specific hydroelastic problems of wave-bed-structure interaction even more complex, and strengthen the interest in development of methodologies to better predict their effects.

Figure 2. Emergency mobile unit in Tokyo gulf (left). Prototype for installation of offshore wind farm in open sea (right)

Although non-linear effects are of specific importance, as, e.g., in the study of significant local slamming phenomena, (Faltinsen, 2001, Greco *et al*, 2003), still the solution of the linearised problem provides valuable information, serving also as the basis for the development of weakly non-linear models. The linearised problem associated with the hydroelastic responses of VLFS can be effectively treated in the frequency domain, and many methods have been developed for its solution. Similar techniques have been developed for the interaction of water-waves with ice sheets; see Fig.3. In the case of water-wave interaction with semi-infinite ice sheets, Balmforth & Craster (1999) used a Fourier transform approach in conjunction with Wiener-Hopf techniques, Linton & Chung (2003) developed a residue calculus technique, and Evans & Porter (2003) used eigenfunction expansion methods to study wave scattering by narrow cracks in ice sheets. A more thorough review concerning wave-ice interaction can be found in the above papers.

In most works dealing with the hydroelastic analysis of large floating bodies, the water depth has been assumed to be constant, either finite or infinite. This assumption cannot, in general, be justified in the case of (very) large floating bodies in nearshore and/or coastal waters. In this case, the variations of bathymetry over the extent of the floating body may be significant and might have important effects on the hydroelastic behaviour of the system; experimental evidence of the effects has been provided by Shiraishi *et al* (2003).

Figure 3. Floating dock in Ujina port, Japan (left). Ice sheets of general finite thickness under the action of waves (right).

The key objective of the proposed project is the development and improvement of computational tools that permit the simulation of the hydrodynamic-hydroelastic interaction of sea waves with large floating structures with general characteristics in areas of general 3D bathymetry, in order to assist both in the further development of such floating structures (as explained in more detail below), and in the prediction of ice detachment processes and the boundary between sea and ice. Below, the main research areas of the proposed project are analysed, with the implementation methodology of the research activities being distributed across the various work packages.

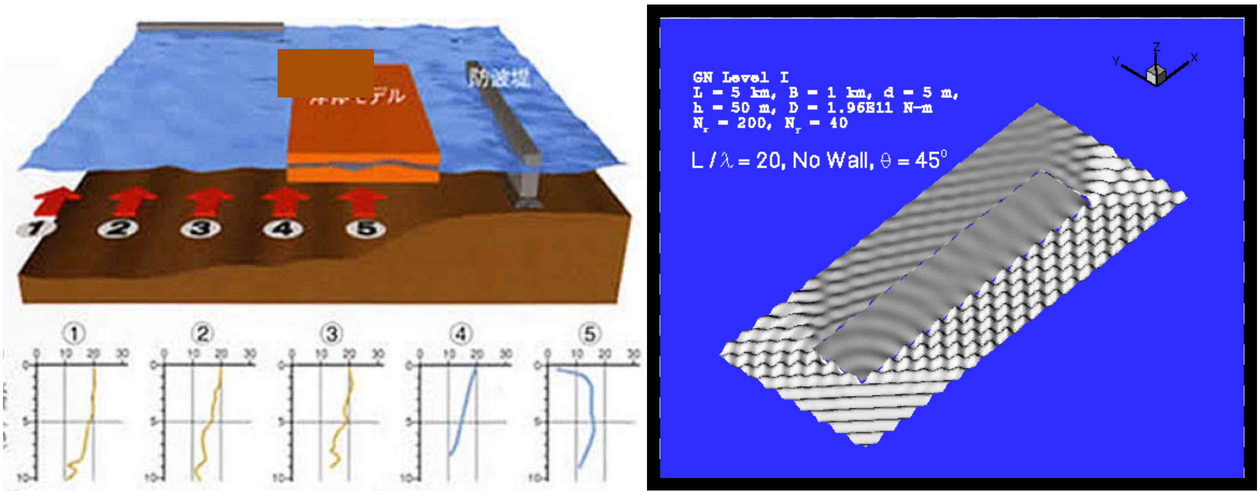

Figure 4. Hydrodynamic interaction of waves and large floating structure in general bathymetry (left). Prediction of elastic response of thin floating plate under the action of surface gravity waves (right).

WORKPACKAGES AND METHODOLOGY

WP1. Development and optimisation of specific models and numerical techniques for the description of wave propagation over coastal and marine areas of varying 3D bathymetry.

Description: The purpose of this WP is the development and optimisation of a reliable wave model, which satisfactory describes surface water waves propagating from offshore to coastal areas; these include calculations throughout the flow field (i.e. calculations from the bed up to the water surface water column), without the presence of structures. This provides the basis for stating and solving the mathematical hydroelastic problem which will then include the effect of the VLFS; with diffraction being its main characteristic. This WP involves the following activities:

1. Development and optimisation of a coupled-mode model, capable of describing the entire wavefield in detail without the presence of structures over general varying 3D bathymetry.
2. Numerical solution of the wave problem, based on the above model, for periodic waves, in various selected conditions and wavefields (wave frequencies, bathymetry).
3. Comparisons with simpler models (Boussinesq type), and evaluation of this model in varying general 3D bathymetry. Comparisons with field measurements and model calibration.

WP2. Hydroelastic analysis of very large floating bodies (VFLS) in a general ocean/coastal fields within the context of water-wave and thin-elastic-plate theory.

Description: In the present WP a continuous, nonlinear, coupled-mode technique is developed and applied to the hydroelastic analysis of very large floating structures of shallow draft over a general bottom topography, based on the coupled-mode model developed by WP1, for waves propagating in variable bathymetry regions. Under the assumption of small deflections and neglecting the rotation of plate section, the shallow-draft platform will be modeled as a thin floating plate, using linear elastic plate theory. This model will be the basis for comparisons for more sophisticated models that are going to be developed in the following WPs. This WP involves the following activities:

1. Theoretical expression of the combined hydrodynamic-hydroelastic problem, in wavefields of general bathymetry, in the context of thin-elastic-plate theory with general characteristics of mass and stiffness.
2. Development and numerical solution of the above problem with an extension of the coupled-mode model of WP1.
3. Comparisons with simpler models that correspond to simplified environments, such as constant water depth, and evaluation of the presented model in general, varying 3D bathymetry.

WP3. Development of improved theory (in vacuo) of finite thickness plates, including shear stress effects confined to general/variable thickness plates with the presence of layers.

Description: Development of a new model for shear deformable plates (or beams), derived by an enhanced representation of the elastic displacement field. The present model contains additional elastic vertical modes, permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate. This model will extend third-order plate theories by Reddy (1984) and Bickford (1982) (see also Wang, Reddy & Lee 2000, Kokkinos 2010, 2002) to plates and beams of general shape.

1. Development of a new model for shear deformable plates based an enhanced representation of the elastic displacement field. The present model will contain additional elastic vertical modes, permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate.
2. Equivalent reformulation in the form of coupled system of differential equation with respect to the amplitude of the elastic modes.
3. Numerical results. Examination of various test cases and comparison with simpler models based on thin elastic plate theory.

WP4. Development and application of a novel coupled-mode model for the VLFS responses with the use of an improved plate theory. Applications in floating and submerged, long bodies, under the impact of the local wavefield.

Description: The purpose of this WP is the conjunction between the coupled-mode model of WP1 with the floating elastic body in a marine/coastal region (characterised by general bathymetry), modeled on the context of improved elastic-plate theory of finite thickness, with general borders, with the possible presence of internal discontinuities/layers and with general characteristics of mass and stiffness (per unit of surface area) that will be produced in WP3. This WP involves the following activities:

1. Theoretical expression of the combined hydrodynamic-hydroelastic problem, in wavefields of general bathymetry, in the context of the improved theory of shear deformable elastic plates of finite thickness, with general characteristics of mass and stiffness.
2. Development and numerical solution of the above problem with an extension of the coupled-mode model of WP2.
3. Comparisons with the simpler model described in WP2 and evaluation of the presented model in general, varying 3D bathymetry. Comparisons with experimental data and demonstration of the dominance of the novel model of this WP.

WP5. Application of finite element method for the examination of local phenomena. Global-local model conjugation including coupled-mode types.

Description: New computational (FEM) structural models will be developed and tested for Floating Structures, simulation algorithms for the prediction of the sea-ice interaction phenomena, and contribution in the assessment of worldwide environmental effects. This WP involves the following activities:

1. Development of approximation techniques (FEM models) in order to evaluate the local dynamic behaviour and assess the strength of critical structural components of VLFS.
2. Numerical results for the local models (stresses, strains, stress intensity factors, etc) and comparison with experimental results and data from the international literature.

WP6. Evaluation and assessment of results – publicity actions.

Description: This WP involves the following activities:

1. Results Evaluation: The assessment of the proposed project will be carried out on a regular basis with the cooperation of all members of the research team and external partners. Core activity in the last WP is the viability study and the possibility of extending the cooperation beyond the end of the proposed project. Also, the head of the project will prepare the progress report where annual reviews, as well as the data on output indicators, will be recorded. Two months prior to the formal completion of the project the Main Research Team will produce an internal evaluation report, which will be targeted both on the positive results of the overall project and possible weaknesses and problems encountered during its implementation project.
2. Publicity actions: Within this action are included: the production of information material associated with the objects of the proposed research project, the development of a relevant webpage in which additional information on the progress and results of the project, as well as the technical reports and published papers will be uploaded.
3. Seminar organisation: Finally, the organisation of specialised seminars is proposed with major participation of Professor J.N. Reddy from the University of Texas A&M and Professor G. Athanassoulis from the NTUA; being enriched by the assistance of the other members of this research group. The first meeting, which will last a week and the main speaker will be Professor J.N. Reddy, focuses on variational pronciples and the mathematical foundation of finite element methods, with applications in engineering, especially in terms of dynamic analysis of complex elastic plates, in general positioning, but also in relation to the objective of this project. It is noted that Prof. J.N. Reddy is an expert of international standing in the above issues, with a wide research work and a rich publication list, the last forty years and his participation in this project is considered particularly honorary by our research group. The second seminar, will also last a week with the main speaker being Prof. G. Athanassoulis, focuses on issues of maritime hydrodynamics and hydroelastic analysis of floating structures. Also, the participation of qualified researchers from the NTUA and the Institute of Oceanography, part of the Hellenic Centre for

Marine Research, will demonstrate the interest and the interface of the problems with subjects of production of marine technology and protection of the coastal area.

Progress beyond the state of the art: As already mentioned, the basic objective of the proposed project is the production and improvement of computational tools that permit the simulation of hydrodynamic-hydroelastic wave-structure interactions, concentrating in large floating elastic bodies, in seas of general characteristics, in offshore and near-shore areas. The above problem becomes even more complicated in environments of varying 3D topography. Indeed, in near shore areas phenomena as refraction, diffraction and shoaling makes the wave-structure interaction problem even more difficult. This project will deal with the above issues by developing innovative nonlinear mathematical models in order to describe wave propagation over varying general bathymetry including the wave interaction with large floating structures; the latter not being dealt before. This will allow the development of new technological processes in the design and the production of such structures, including the study of the impact on the coastal zone, as well as the wave-ice interaction, which is a major environmental problem particularly in Polar Regions with global implications.

Expected benefits in local and international level: The proposed research contributes to the technology for large floating structures, which can be used in Greece within the coastal zone for the economic development of islands in the Aegean and Ionian seas. Apart from that, VLFS is the main infrastructure for the development of maritime and offshore wind energy through offshore wind farms. EU Commission considers this the energy of the future, as it represents a source of clean, indigenous and renewable energy (its utilisation will be 40 times greater by 2020). Benefits: the production units at sea are larger than on land, winds are stronger and more stable at sea, wind farms protect certain marine ecosystems and provide new uses of the sea (offshore aquaculture benefiting from the VLFS). It is necessary to have the appropriate technology and scientific resources to fully exploit VBLFS and, in this perspective, to develop synergies between clean energy production and maritime technology.

The proposed work aims the creation and strengthen of a research team in TEI-Athens, in collaboration with the National Tech. Univ. of Athens, the Hellenic Center for Marine Research, the Dept. of Mech. Engineering at Texas A&M Univ. and others external associates, with mean activity in the development and optimization of large elastic floating structures technology that are located and operating in the near-shore and coastal environment. Also the proposed work, through the collaborations and the final deliverables serves as an integration of a long term research of the various members in various research directions.

Additionally, an important goal of the work is the optimization of experimental measurements using the small-scale towing tank of the Naval Architecture Dept. and the improvement of the prediction abilities of the experimental procedure through the assimilation of analytic models. For supporting the conduction of the measurements of hydrodynamic responses of elastic floating bodies to waves, the commission of special measurement devices is proposed (e.g. electronic accelerometers) and software for data acquisition is proposed. Thus the proposed work will complement the existing installed measurement equipment highlighting and extending its capabilities.

STATE OF THE ART REVIEW AND LITERATURE SURVEY

Free-surface gravity waves over variable sea-bottom topography: The design of coastal structures and the prediction of the evolution of the coastal environment require detailed information about inshore wave conditions. In many applications this becomes possible only by transformation of offshore sea states, since information from global forecasting models, satellite measurements or buoy measurements is practically available offshore (usually at a distance of several tenths of kilometres far from the coast). One approach to treat this problem is to apply shallow-water wave models to transform offshore wave directional spectra to inshore spectra, exploiting the available geographical information (bathymetry, coastline); see, e.g., Goda (2000, Ch.3). The same approach has been also very successfully applied to derive local wave climatology in nearshore areas along the European coast from offshore time series of historical (hindcast) wave data; see, e.g., Athanassoulis *et al.* (1999).

Shallow-water wave models are classified into the families of phase-averaged and phase- resolving models, in accordance with the rate of spatial evolution of the wave field, Battjes (1994). In the first family the variation of the local wave properties in the scale of the mean wavelength is assumed small and, thus, phase-averaging can be applied, while in the second family, the local wave properties are assumed to vary within distances of the order of the mean wavelength. The wave spectrum itself is a phase-averaged quantity, and thus, it is most convenient to deal with it with a phase-averaged model, Battjes (1994). Phase-averaged models can take into account most of the significant shallow-water processes, e.g., depth refraction and shoaling, current refraction, quadruplet and triad interactions, wind input, whitecapping, depth-induced breaking and bottom friction, Booij *et al.* (1999), Ris *et al.* (1999). On the other hand, diffraction, being important in the case of wave-bottom interaction in the presence of abrupt bathymetric variations in shallow water and in the near field of wave-structure interactions, necessarily requires the application of phase-resolving models.

Models of the phase-averaged category have been already converged to a single basic archetype-model, (Battjes, 1994), and are based on the solution of the radiative transfer equation, expressing the spectral wave energy or the wave action balance, (Komen *et al.*, 1994, Massel, 1996). This kind of equations form the basis of today's most advanced wave forecasting models. Older Lagrangian or ray-theory techniques (as, e.g., Cavaleri and Malanotte- Rizzoli, 1981) for approximately solving the radiative transfer equation have been surpassed by more efficient third-generation models, based on fully numerical schemes for solving the energy balance equation in Eulerian coordinates. The main drawback of this approach in coastal-water applications is the inability to deal with diffraction.

Phase-resolving models are generally based on mass and momentum balance equations and can fully treat the diffraction effects. These models can be further distinguished into classes depending on the exactness of the representation of the vertical structure of the wave-field. If the depth is assumed arbitrary but slowly varying, and the free-surface condition is linearised, by using a perturbation technique, a class of mild-slope models and their parabolic approximations are obtained, see, e.g., Li *et al.* (1993), Suh *et al.* (1990), Suh and Dalrymple (1993). If relative depth and bottom slope are assumed small, and the non-linearity weak, Boussinesq models apply. Improved versions of these models with enhanced dispersion characteristics, extending the range of applicability to larger depths and/or variable bathymetry, have been reported by many authors; see, e.g., Madsen and Sorensen (1992), Nwogu (1993), Liu (1995), Kirby (1997), and the references cited therein. In general, however, these models cannot be relied on as the depth increases, or the bathymetry is not slowly varying. Another important class of models are the ones considered as weakly non-linear generalisations of the mild-slope equation, as e.g., the models developed by Beji and Nadaoka (1997), Nadaoka *et al.* (1997), Tang and Quellet (1997). These models can describe combined refraction-diffraction of weakly non-linear water waves, but still suffer from the assumption of slowly varying bathymetries. An essential limitation of phase-resolving models comes from the space and time resolution requirements of these models, restricting their practical applicability to regions with dimensions of the order of several tenths of wavelengths (say of the order of 1 to 2 km).

General numerical methods for the solution of the linearised water-wave problem in variable bathymetry, such as finite element methods, boundary integral equation methods or hybrid techniques are available, Yeung (1982), Tsai & Yue (1996). However, the computational cost of these generic techniques is high, rendering them inappropriate, especially for long-range propagation and/or in three dimensions. Due to this fact, a constantly growing emphasis has been given on the development of wave models which better exploit the essential features of gravity waves and are better suited for long-range propagation applications.

One very attractive family of models is obtained by reformulating the problem as a system of equations on the horizontal plane with variable coefficients. Berkoff (1972, 1976) derived a one-equation model for gentle bottom slopes, called the mild-slope equation, in which the vertical distribution of the wave potential has been prescribed. Other derivations of similar or improved one-equation models, using either averaging techniques or variational principles, have been given by Smith & Sprinks (1975), Booij (1981), Radder & Dingemans (1985), Kirby (1986a, b), Massel (1993); see also the general surveys by Mei (1983), Massel (1989), Porter & Chamberlain (1997) and Dingemans (1997). In general, mild-slope equations can be considered satisfactory for mean bottom slopes up to 1:3 (Booij, 1983, Berkhoff et al., 1982) and some of them can also predict the high backscattering due to Bragg resonance, occurring when an undulating component is superimposed on a slowly varying bottom topography.

The basic restriction inherent to any one-equation model is that the vertical structure of the wave field is given by a specific, preselected function. This restriction makes them inappropriate to describe the wave field when the bottom topography is not slowly varying and the depth is sufficiently small so that the wave strongly interacts with the bottom. The improvement of the mild-slope models to match the requirements of this situation calls for a more general representation of the vertical structure of the wave field. Massel (1993) and Porter & Staziker (1995) presented this kind of models, called extended mild-slope equations, in which the vertical profile of the wave potential at any horizontal position is represented by a local-mode series involving the propagating and all evanescent modes. Then, using either a Galerkin approach (Massel, 1993) or a variational principle (Porter & Staziker, 1995), an infinite set of coupled equations for the unknown amplitudes is obtained, which is called the coupled-mode system. However, this expansion has been found to be inconsistent with the Neumann condition on a sloping bottom, since each of the vertical modes involved in the local-mode series violates it and, thus, the solution, being a linear superposition of modes, behaves the same. This fact has two important consequences. First, the velocity field in the vicinity of the bottom is poorly represented and, secondly, wave energy is not generally conserved. This problem has been remedied by the consistent coupled-mode theory recently developed by Athanassoulis & Belibassakis (1999). In the latter model the standard local-mode representation is enhanced by including an additional term, called the sloping-bottom mode, leading to a consistent coupled-mode system of equations. This model is free of any simplifications concerning the vertical structure of the wave field and of any assumptions concerning the bottom slope and curvature, and it is consistent since it enables the exact satisfaction of the bottom boundary condition and the calculation of bottom velocities. Finally, it has been extended to treat wave propagation and diffraction in general three-dimensional environments, Belibassakis *et al* (2001), and to predict second-order waves in variable bathymetry, Belibassakis & Athanassoulis (2002).

It is well known that practical and realistic results concerning the wave motion and its impact on coastal and surf zones can be obtained by including dissipation of energy due to bottom friction and wave breaking. The inclusion of these effects in the modified mild-slope equation has been presented by Massel (1992, 1996), by introducing appropriate damping terms. The corresponding extensions of the above coupled-mode system to include dissipation effects have been presented in Belibassakis et al (2007, 2008). Furthermore, application to the transformation of wave systems characterized by a directional spectrum over large three-dimesional nearshore and coastal areas is presented in Gerostathis et al (2008), and further extensions to treat wave-seabed-current interaction in variable bottom topography are presented in Belibassakis et al (2007, 2008, 2010).

In the last years several mathematical models and approximation techniques have been developed to treat the fully non-linear problem of water waves propagating in variable bathymetry regions at various degrees of approximation. An important feature of this problem is that propagation phenomena take place on the horizontal plane, and non-local couplings (wave-wave and seabed-wave) exist through the vertical structure of the flow field. Fully numerical methods, based on FEM (e.g., Wu & Eatock Taylor 1994, 1995, Turnbull et al 2003), Finite Difference Methods, in conjunction with vertical transformation (e.g., Bingham & Zhang, 2007, Engsig-Karup et al, 2009), spectral methods (Dommermuth &Yue, 1987, Bateman et al, 2001), BEM (e.g., Grilli et al 1989, 2001, Ohyama & Nadaoka 1991, 1994, Christou et al 2009), and Smooth Particle Methods (e.g., Dalrymple & Rogers 2006, Dalrymple et al 2007) have been developed to treat the problem in general bathymetry domains; see also the reviews by Tsai & Yue (1996) and Dias & Bridges (2006). The above methods based on BEM and SPH are able to represent highly nonlinear phenomena, as wave folding and breaking, but they are computationally intense and thus, their use is more appropriate for short-range water wave propagation and application to local interaction problems.

Various equivalent reformulations of the fully nonlinear-nonlocal water-wave problem, in finite water depth, have been obtained, as e.g., by means of Hamilton's principle and the Dirichlet to Neumann (DtN) map (Craig & Sulem, 1993), by means of Lagrange equations of fluid dynamics and analyticity of the wave potential in the liquid domain (e.g., Craig 1985, extended by Wu 1999 in the 3D case, using Clifford analysis), variational principles (e.g., Groves & Toland 1997) and other approaches. In particular, using as canonical variables the values of the wave potential on the free-surface and the free-surface elevation $\varphi\left(x,t\right)=\Phi\left(x,z=\eta\left(x,t\right),t\right)$ and $\eta=\eta\left(x,t\right)$, in conjunction with Hamilton's principle, the following system is obtained in 2D (see also Craig & Sulem 1993),

$$\eta_t - G\left(\eta\right)\circ\varphi = 0, \qquad \varphi_t + \frac{1}{2\left(1+\eta_x^2\right)}\left(\varphi_x^2 - \left(G\left(\eta\right)\circ\varphi\right)^2 - 2\eta_x\varphi_x G\left(\eta\right)\circ\varphi\right) + g\eta = 0, \tag{1}$$

where $G\left(\eta\right)\circ\varphi = \left(1+\eta_x^2\right)^{1/2}\left[\partial\Phi / \partial n\right]_{z=\eta\left(x;t\right)}$ denotes the DtN operator ($\partial\Phi / \partial n$ is the normal derivative), which is defined through the b.v.p on Φ, consisted by the Laplace equation, the bottom (no-entrance) boundary condition, the Dirichlet data on the instantaneous free-surface $\eta=\eta\left(x,t\right)$ and appropriate lateral conditions. These nonlinear, nonlocal equations have been extensively studied, in constant depth (see, e.g., Craig & Sulem, 1993, Craig & Nicholls, 2002) and in variable bottom topography, (e.g., Craig et al, 2005). For small (but finite) free-surface elevation, the nonlocal operator $G\left(\eta\right)$ is analytic (Coifman & Meyer, 1985), and thus, it can be expanded in functional Volterra-Taylor series. In constant depth, the successive terms are obtained recursively (Craig & Sulem 1993). For treating the problem in general bathymetry domains, the DtN map could be based on a boundary integral formulation (see, e.g., Clamond & Grue, 2001, Grue, 2002). On the other hand, various simplified formulations (leading to model equations) of the water-wave propagation problem have been obtained by many authors under various asymptotic assumptions. In this direction, we mention here the weakly nonlinear system with enhanced dispersion characteristics by Trulsen & Dysthe (1996), Trulsen et al (2001). Another, simple model equation, retaining up to a degree nonlinearity and nonlocality is Witham's equation (1967). The generalised Witham system (see, e.g., Naumkin & Shishmarev 1994) combines explicit (quadratic) nonlinearity with linear nonlocal terms, modelling to some extent dispersion. From the latter system, Witham and Boussinesq model equations can be obtained as special cases. Applications of the Boussinesq model to water wave propagation and interaction with the seabed in coastal areas are numerous. We mention here specific extensions of this model to the case of mildly varying bathymetry by Peregrine (1967), for which numerical solvers have been presented by various authors; see, e.g., Beji & Battjes (1994). Extended Boussinesq systems fully accounting for nonlinearity and dispersion have been also developed; see, e.g., Liu (1995), Madsen et al (2002, 2003). Similarly, Green-Naghdi models (e.g., Kim et al 2001, 2003) have been developed and successfully applied to nonlinear water waves in variable depth.

The usual form of Bousinessq models is with respect to free surface elevation η and horizontal velocity u, however, they can be also formulated with respect to the free surface elevation and wave potential φ (Massel 1989, Chap.4), as follows:

$$\eta_t + \Big(\big(h+\eta\big)\varphi_x \Big)_x = 0, \qquad \varphi_t + g\eta + \frac{1}{2}\big(\varphi_x\big)^2 - \frac{h^2}{3}\varphi_{xxt} + ... = 0, \tag{2}$$

where h denotes the (possibly varying) depth. In the same direction, extended versions of mild-slope equation have been presented by various authors with applicability also to short wave–bottom interaction problems, e.g., Beji and Nadaoka (1997), Nadaoka et al (1997), Tang and Ouellet (1997). Also, high-order methods based on the velocity potential formulation have been developed by Jamois et al (2006), Bingham et al (2009) and others.

In constant, finite water depth nonlinear wave solutions have been obtained, both analytically (see, e.g., Fenton 1990) and numerically (e.g., Schwartz 1974, Cokelet 1977, Rienecker & Fenton 1981, Drennan et al 1988 and others). A detailed presentation of nonlinear steady water waves, including the effects of surface tension, and discussing in detail the appearance turning points and bifurcation of highly nonlinear solutions, can be found in Dias & Kharif (1999) and in Okamoto & Shoji (2001). In particular, Schwartz (1974) and Cokelet (1977) extended the Stokes theory to very high order, calculating the speed, momentum, energy and other integral properties of waves in water of arbitrary uniform depth. Parallel to Stokes expansion, nonlinear methods based on the use of truncated Fourier expansions have been also developed, see, e.g., Rienecker and Fenton (1981). The latter approaches are based on stream function formulation, however similar developments based on wave potential have been presented. In this direction, a homotopy analysis method based on the velocity potential formulation is more recently presented by Tao et al (2007) to describe the nonlinear progressive waves in water of finite constant depth. Fourier expansion methods have been also proved very useful tools for the description of nonlinear transient wave systems, including application to irregular wave kinematics (see, e.g., Sobey 1992, Baldock & Swan 1994, Johanessen & Swan 1997).

In Belibassakis & Athanassoulis (2011), the problem of non-linear gravity waves propagating over a general bathymetry in non-uniform domains characterised by variable bottom topography is considered. An essential feature of this problem is that the wave field is not spatially periodic. Extra difficulties are introduced by the fact that no asymptotic assumptions concerning the free-surface and bottom slope are made. An alternative and efficient, from the point of view of the numerical solution, reformulation is made in the form of a system of horizontal differential equations governing the evolution of water waves in variable bathymetry regions. This formulation is equivalent with the fully nonlinear water-wave problem without introducing any simplifying assumptions concerning the characteristic non-dimensional parameters, and belongs to the same category with high-order models based on velocity potential. Using Luke's (1967) variational principle, in conjunction with an enhanced local-mode series expansion of the wave potential developed by the authors (Athanassoulis & Belibassakis 2002, Belibassakis & Athanassoulis 2006), which has the general form

$$\Phi\big(x,z,t\big) = \sum_{n=-2}^{\infty} \varphi_n\big(x,t\big)\, Z_n\Big(z;h\big(x\big),\eta\big(x,t\big)\Big) \ , \tag{3}$$

where $Z_n\Big(z;h\big(x\big),\eta\big(x,t\big)\Big)$ denote vertical functions that are parametrically dependent on the bathymetry and the free-surface elevation, having the property to form a basis in the vertical interval from the bottom to the free-surface: $-h\big(x\big) < z < \eta\big(x,t\big)$. The above local-mode series contains the usual propagating and evanescent modes, plus two additional terms, the free-surface mode and the sloping-bottom mode, enabling to consistently treat the non-vertical end-conditions at the free-surface and the bottom boundaries and exhibiting fast convergence. The coupled-mode system is formulated as a set of two evolution equations on the free-surface wave potential and corresponding elevation $\big\{\varphi,\ \eta\big\}$, as follows:

$$\frac{\partial\eta}{\partial t} + \frac{\partial}{\partial x}\left(\big(\eta+h\big)\frac{\partial\varphi}{\partial x}\right) + N_1\big(\eta,\varphi_n\big) = 0 \ , \quad \frac{\partial\varphi}{\partial t} + g\eta + \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + N_2\big(\eta,\varphi_n\big) = 0 \ , \tag{4}$$

where the nonlinear operators $N_{1,2}\left(\eta,\varphi_n\right)$ are defined in terms of the modal amplitudes. The above coupled-mode system is applied to the numerical investigation of families of steady travelling wave solutions in constant depth regions, corresponding to a wide range of water depths, ranging from intermediate to shallow wave conditions. The derivation of the latter solutions is important for comparison with known theories and validation of the present model, as well as for the consistent formulation of boundary conditions for the CMS in the case of water wave propagation in non-homogeneous environments. Numerical results are compared vs. Stokes and cnoidal wave theories, as well as vs. fully nonlinear Fourier methods, respectively, showing very good agreement. It is illustrated that the present coupled-mode system fully accounts for the effects of non-linearity and dispersion, and the local-mode series exhibits fast convergence. Thus, a small number of modes (up to 5-6) are usually enough for the precise numerical solution, provided that the two new modes, the free-surface and the sloping-bottom ones, are included in the local-mode series (the latter only in the case of non-horizontal bed). Finally, numerical results are presented for waves propagating over variable bathymetry regions and compared with nonlinear methods based on boundary integral formulation and experimental data, showing very good agreement.

In concluding this subsection, we mention that the above coupled-mode model has been extended from members of the present research group to treat obliquely incident waves in three dimensional bathymetry, including diffraction effects from localized three-dimensional scatterers; see Belibassakis *et al* (2001). Similar techniques have been developed by members of the present group for acoustic propagation and scattering in general multilayered waveguides, governed by the Helmholtz equation, with application to ocean and coastal hydroacoustics; see, e.g., Athanassoulis et al (2008). The detailed study and numerical investigation of this system one of the main objectives of this project, including a comparison with simplified models of type equations, Boussinesq (e.g. Belibassakis et al 2004) in shallow water, and the experimental verification and validation in the experiment tank of TEI Athens. The accurate knowledge of the wave field allows the calculation of hydrodynamic behaviour of ships and floating structures operating in marine and coastal environment, the prediction of wave-induced loads and the hydrodynamic responses.

Hydroelastic Analysis of large floating bodies

General methods for the dynamical analysis of such complex fluid - elastic structure interaction problems have been developed, based on variational principles and FEM (see, Xing et al 1996). Taking into account that the horizontal dimensions of the large floating body are much greater than the vertical one, thin-plate (Kirchhoff) theory is commonly used to model this situation. Also, although non-linear effects are of specific importance (Faltinsen 2001), still the solution of the linearised problem provides valuable information, serving also as basis for the development of weakly non-linear models (Belibassakis & Athanassoulis 2006).

The linearised hydroelastic problem is effectively treated in the frequency domain, and many methods have been developed for its solution. These include hydroelastic eigenfunction expansion techniques (Kim & Ertekin 1998, Takagi *et al* 2000, Hong *et al* 2003), Boundary Element Methods (Ertekin & Kim, 1999, Hermans, 2000), B-spline Galerkin method (Kashiwagi 1998), integro-differential equations (Adrianov & Hermans, 2003), Wiener-Hopf techniques, (Tkacheva, 2001), Green-Naghdi models (Kim & Ertekin, 2002), Galerkin methods (Eatock Taylor, 2007) and others. Another approach, originally developed by Eatock Taylor & Waite (1978) and Bishop *et al* (1986), and further extended by various authors, as e.g., Newman (1994) Wu *et al* (1995), is based on expressing the structure oscillations in a series expansion (using either dry elastic modes or another basis), identifying appropriate radiation problems and, finally, formulating and solving the coupled hydrodynamic equations. Moreover, Meylan (2001) derived a variational equation for the plate-water system by expressing the water motion as an operator equation. In addition to the above, high-frequency asymptotic methods have been developed to describe the deflection dynamics of VLFS, see, e.g., Ohkusu & Namba (1996), Hermans (2003). The latter are especially useful in the case of short waves interacting with a floating structure of large horizontal dimensions.

Similar techniques have been developed for the interaction of water-waves with ice sheets. For example, Marchenko & Shrira (1991), using Zakharov (1968) variational principle, developed a Hamiltonian formalism for the waves in the liquid beneath an ice sheet, and Meylan & Squire (1994) used Green's function approach to formulate an integral equation over the floating plate.

In the case of an ice sheet of varying thickness, floating on water of varying depth, Porter & Porter (2004) derived a simplified model using a variational principle and invoking the mild-slope approximation with respect of the ice thickness and water depth variations. Numerical methods for predicting the linearised hydroelastic responses of VLFS in variable bathymetry regions have been also developed, based on BEM (Utsunomiya *et al* 2001, Wang & Meylan 2002), FEM (Kyoung et al 2005), on eigenfunction expansions in conjunction with step-like bottom approximation (Murai et al 2003). In the case of the hydroelastic behaviour of large floating bodies over general bathymetry, modelled as thin homogeneous plates, a coupled-mode system has been derived and examined by the authors (Belibassakis & Athanassoulis 2005). This method is based on a local vertical expansion of the wave potential in terms of hydroelastic eigenmodes, extending previous similar approach for the propagation of water waves in variable bathymetry regions (Athanassoulis & Belibassakis 1999, Belibassakis *et al* 2001). A similar approach based on multi-mode expansion has been presented by Bennets et al (2007), with application to wave scattering by ice sheets of varying thickness. The wave potential in the water column is represented by means of a local mode series expansion containing an additional mode providing appropriate correction term on the bottom boundary, when the slope is not mild. In the above sense, the present method extends previous approaches concering free-surface hydrodynamic and hydroelastic problems based on thin plate theory (Porter & Porter 2004, Belibassakis & Athanassoulis 2005, 2006). In the simple two-dimensional (horizontal-vertical) case the local mode expansion has the form:

$$\varphi\left(x,z\right)=\varphi_{-1}\left(x\right)Z_{-1}\left(z;x\right)+\sum_{n=0}^{\infty}\varphi_{n}\left(x\right)Z_{n}\left(z;x\right),\ -h\left(x\right)<z<-b\left(x\right), \tag{5}$$

where $Z_n\left(z;x\right)=\cosh^{-1}\left(\kappa_n H\right)\cosh\left[\kappa_n\left(z+h\left(x\right)\right)\right]$ are local vertical eigenmodes from the bottom boundary to the top one derived as solution of Strurm-Liouville vertical eigenvalue problems, and the corresponding eigenvalues $\left\{\kappa_n, n=0,1,2...\right\}$ are obtained as roots of the dispersion relation:

$$\mu H=\alpha\left(\kappa\right)\ \kappa H\tanh\left(\kappa H\right)\quad, \tag{6}$$

where $\mu=\omega^2/g$ is the frequency parameter. The above dispersion relation with $\alpha=1$ reduces to the linear dispersion relation of water waves under the free surface and with $\alpha\left(\kappa\right)=D\kappa^4+1-\varepsilon$ to the classic hydroelastic dispersion relation under a thin, floating elastc plate, where D is the flexural rigidity parameter and ε the corresponding mass parameter (per unit horizontal surface) of the floating plate. Using Eq. (5) in the variational principle (Athanassoulis & Belibassakis 2009) leads to the following horizontal system of coupled differential equation with respect to the modal amplitudes φ_n:

$$\sum_{n=-1}^{\infty}a_{mn}\left(x\right)\frac{\partial^2\varphi_n}{\partial x^2}\left(x\right)+b_{mn}\left(x\right)\frac{\partial\varphi_n}{\partial x}+c_{mn}\left(x\right)\varphi_n\left(x\right)=\ i\omega w\left(x\right),\quad m=-1,0,1,\ldots., \tag{7}$$

where *a,b,c* are coefficients defined through the local vertical eigenmodes $Z_n\left(z;x\right)$. The second-order system (7) is further coupled with the following fourth-order equation involving the local vertical deformation *w* of the elastic floating body

$$\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)+\left(1-\varepsilon\right)w=\frac{i\mu}{\omega}\sum_{n=0}^{\infty}\varphi_n\left(x\right)\quad, \tag{8}$$

based on first-order (Kirchhoff) plate theory. We remark here that the fast convergence property of the above local-mode series facilitates the modelling of floating, shear deformable bodies, with variable thickness, elastic parameters and mass distribution, including applications to the problem of wave interaction with ice sheets of general morphology, including fully 3D problem of finite rafts over a general seafloor, and the examination of the effects of non-linearity.

A significant improvement of plate (and beam) theory is provided second (e.g. Mindlin, 1951) and higher order models permitting also to introduce shear effects into consideration. In particular, third-order models, involvein general five 2D fields (or nine in 3D plate bending). Among the various third-order models appearing in the literature (reviewed by Wang et al. 2000 and Reddy 2004), perhaps the most interesting one is that developed by Bickford (1982) and Reddy (1984b), based on the following displacement expansion:

$$u_x\left(x,z;t\right)\ =\ u_0\left(x;t\right)+z\,u_1\left(x;t\right)+\left(-\frac{4}{3h^2}\right)z^3\left(u_1+\frac{\partial w_0}{\partial x}\right),\ u_z\left(x,z;t\right)\ =\ w_0\left(x;t\right). \tag{9}$$

The above representation of the displacement field clearly accommodates for a quadratic variation of transverse shear strains and stresses, vanishing at the top and bottom surfaces of a plate, which is the correct boundary condition violated by all lower (than three) order plate theories. Concerning the satisfaction of the latter condition, the Reddy-Bickford model is consistent only for plates with essentially horizontal boundaries. In the case of more general boundaries, like the lower surface of the floating plate modelling the thick ice sheet in Fig.1, an additional fourth-order term (or plate mode) is required for the consistent satisfaction of zero shear stress, quite similarly as the introduction of the additional mode was necessary for the consistent satisfaction of the bottom boundary condition by the local-mode expansion Eq. (5) in the case of general bathymetry (see discussion in Athanassoulis & Belibassakis 1999). The consistent representation leading to an enhanced higher-order plate/beam model will be presented in more detail in the next subsection.

In Athanassoulis & Belibassakis (2009) the problem of hydroelastic analysis of a thick, nonuniform, shear deformable floating elastic body, lying over a variable bathymetry regions is addressed, with application to scattering of coupled, hydroelastic waves propagating through an inhomogeneous sea-ice environment, containing ice sheets of variable thickness and non-mildly sloped interface. A new coupled-mode system of horizontal equations is presented, based on the theory of shear deformable plates (or beams), derived by an enhanced representation of the elastic displacement field. This model contains additional elastic vertical modes, permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate. This model extends third-order plate theories by Reddy (1984) and Bickford (1982) (see also Wang et al 2000) to plates and beams of general shape. The present coupled-mode system of horizontal differential equations is obtained by means of a variational principle composed by the one-field functional of elastodynamics (see, e.g., Graff 1975, Reddy 1984) in the plate region, and the Luke's (1967) pressure functional in the water region. In this case the 4-order horizontal differential equation for the vertical deflection takes the form

$$\frac{\partial^2}{\partial x^2}\left(De_\nu\left(1-\delta\right)\frac{\partial^2 w}{\partial x^2}\right)-\varepsilon\frac{b^2}{12}\frac{\partial^2 w}{\partial x^2}+\left(1-\varepsilon\right)w=\frac{i\mu}{\omega}\sum_{n=0}^{\infty}\varphi_n\left(x\right)\quad, \tag{10}$$

where e_ν and δ are appropriate parameters involving the shear modulus and Poisson's ratio, and $b(x)$ the finite and variable thickness of the floating elastic plate. We see that in this case the function α involved in the dispersion relation, Eq. (6) of the vertical eigenmodes in the water column below the floating plate, is generalized as follows:

$$\alpha\left(\kappa\right)=De_\nu\,\kappa^4\left(1-\delta\right)+1-\varepsilon\left(1+\frac{\kappa^2 b^2}{12}\left(1-\delta\right)\right), \tag{11}$$

which, is a consistent extension of the classical first-order hydroelastic coupled-mode theory (Porter & Porter 2004, Belibassakis & Athanassoulis 2005) having the asymptotic property to reduce to it in the case of infinitesimal (thin) plate thickness ignoring the shear effects.

Variational Principles of Continuum Mechanics and Finite Element Methods with applications in the local dynamic analysis of Structures

The Finite Element Method (FEM), Oden & Reddy (1976), Hughes (2000), is a general mathematical technique for the approximate solution of complex problems. The loading conditions and associated input data of the local FEM models (in the time or frequency domain) will be supplied by the general dynamic analysis of the larger hydrodynamic models, see **WP 2** and **WP 4.** The results of the local models will be used in order to optimize the structural behaviour of the larger structure (feedback procedure).

In the current research action, the p-version FEM will be employed, Szabo & Babuška (1991). Emphasis will be given in the adoption of effective algorithms for the reduction of the total computational cost (Kokkinos, 2010). For general dynamic structural problems, assuming small spatial derivatives of the velocity field, the virtual work principle is expressed in Euler coordinates as, (Y.C. Fung and P. Tong, 2001)

$$\int_V \sigma_{ij}\delta\varepsilon_{ij}dV+\int_V \rho\frac{\partial^2 u_i}{\partial t^2}\delta u_i dV=\int_V F_i\delta u_i dV+\int_S T_i\delta u_i dS\,,\quad \text{for all}\quad \delta u_i\,, \tag{12}$$

with $\delta u_i=0$ on the Dirichlet boundary. In Eq. (12) u_i is the displacement field, σ_{ij} are the components of Cauchy stresses, F_i the volume body forces, T_i the external surface tractions on the control volume V, ρ the material density, δu_i a virtual displacement field and $\delta\varepsilon_{ij}$ the small strain field corresponding to the virtual displacements δu_i. For small displacements and linear stress-strain relation, the standard FEM approximation of (8) leads to the following coupled system of ordinary differential equations,

$$\mathbf{M}\frac{d^2\mathbf{u}}{dt^2}+\mathbf{Ku}=\mathbf{F}\ \text{(plus initial conditions at}\ \ t=0\,\text{)}, \tag{13}$$

where $\mathbf{u}\left(t\right)$ is the nodal displacement vector, $\mathbf{M}$ the mass matrix, $\mathbf{K}$ the stiffness matrix and $\mathbf{F}\left(t\right)$ the load vector. The system (9) is solved either by time integration (time domain solution) or by transference into to the frequency domain (modal analysis). The eigen-values ω (and respective eigenvectors) of the discrete FEM model are evaluated based on the well known equation,

$$\det\left(\mathbf{K}-\omega^2\mathbf{M}\right)=0\,. \tag{14}$$

Experience of the present research group in related research. Past collaborations

The proposed project begins by hand, helps to further deepening of the members of this research group's research directions in research and scientific areas such as mathematical modeling of waves in an environment variable bathymetry, the mathematical modeling of the interaction of floating bodies and the effects ripple flexibility and develop shipbuilding technology, on the other hand it leads to significant production of final results (deliverables).

As is apparent from the preceding description, this project will contribute to the joint conclusion a long research project, parts of which have been developed in the past, regardless of any one of the partner groups - researchers in different scientific areas, such as in the mathematical modeling the spread of sea waves in the marine coastal environment, especially in matters of engineering and flexibility of naval hydrodynamics, and the development of shipbuilding technology.

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